lec 3

$L^{-1}\left[\frac{e^{-20s}}{s^2+4}\right]$ $\qquad$ $L^{-1}[e^{-as} f(s)] = u(t-a) f(t-a)$

$= L^{-1}\left[e^{-20s} \overset{f}{\frac{1}{s^2+4}}\right]$ $\qquad$ $f(s) = \frac{1}{s^2+4}$ $\qquad$ $f(t) = \frac{1}{2}\sin 2t$

$= \frac{1}{2} u(t-20) \cdot \sin 2(t-20)$

* *

Prove $L^{-1}[s^k] = 0$ where $k = 0, 1, 2, \dots$

O.D.E

Homogence $\qquad$ non H

① نوجد $\overset{-1}{LT}$ لكل طرف من طرفين

② نفصل الـ Y في طرف وباقي بالطرف الآخر بعد تعويض

itial condition

③ $L^{-1}\, Y(s)$

$L\, y(t) = Y(s)$

$y' = sY(s) - Y(0)$

$y'' = s^2 Y(s) - s y(0) - y'(0)$

$y''' = s^3 Y(s) - s^2 y(0) - s y'(0) - y''(0)$

Ex: Solve the following D.E

$y' - 2y = 5e^{2t}$ $\qquad$ $y(0) = 1$

$sY(s) - y(0) - 2Y(s) = \frac{5}{s-2}$

$sY(s) - 1 - 2Y(s) = \frac{5}{s-2}$

$Y(s)\,[s-2) = \frac{5}{s-2} + 1$

$$y(s) = \frac{1}{s-2} + \frac{5}{(s-2)^2} \qquad y(t) = e^{2t} + 5e^{2t}t$$

Ex $y''' + 2y'' - y' - 2y = 0 \qquad y(0) = 1,\ y'(0) = 2,\ y''(0) = 2$

$$s^3 y(s) - s^2 y(0) - s y'(0) + 2[s^2 y'(s) - s y(0) - y'(0)]$$

$$-[s y(s) - y(0)] - 2y(s) = 0$$

$$y(s) * [s^2 + 2s^2 - s - 2] = s^2 + 2s + 2 + 2s + 4 - 1$$

$$= s^2 + 4s + 5$$

$$y(s) = \frac{s^2 + 4s + 5}{(s-1)(s+1)s+2)}$$

S②

Ex. $y'' - ty' + y = 1$ $\quad y(0) = 1 \quad y'(0) = 2$

$$s^2 y(s) - s \overset{1}{y(0)} - \overset{2}{y'(0)} + \frac{d}{ds}\left(s y(s) - \overset{1}{y(0)}\right) + y(s) = \frac{1}{s}$$

$$s^2 y(s) - s - 2 + [s y'(s) + y(s)] + y(s) = \frac{1}{s}$$

$$s y'(s) + (s^2 + 2)\, y(s) = \frac{1}{s} + (s + 2$$

$$y'(s) + \left(s + \frac{2}{s}\right) y(s) = \frac{1}{s^2} + \frac{s+2}{s} \quad q.x$$

$y' + P(x)y = g(x)$

$\mu = e^{\int P x \, dx}$

$y = \frac{1}{\mu}\left(\int \mu \cdot g(x) + c\right.$

$$\mu = e^{\int (s + \frac{2}{s})\, ds}$$

$$= e^{\frac{s^2}{2} + 2\ln s} = s^2 e^{\frac{s^2}{2}}$$

$$y(s) = \frac{1}{s^2} e^{-\frac{s^2}{2}} \left[\int s^2 e^{\frac{s^2}{2}} \left(\frac{1}{s^2} + \frac{s+2}{s}\right) + c\right]$$

$$y(s) = \frac{1}{s^2} e^{-\frac{s^2}{2}} \left[\int e^{\frac{s^2}{2}} ds + \int s^2 e^{\frac{s^2}{2}} + \int 2 s e^{\frac{s^2}{2}} + c\right]$$

$$\downarrow$$

$$s e^{\frac{s^2}{2}} - \int e^{\frac{s^2}{2}} ds$$

$$= \frac{1}{s^2} e^{-\frac{s^2}{2}} \left[\int e^{\frac{s^2}{2}} ds + s e^{\frac{s^2}{2}} - \int e^{\frac{s^2}{2}} ds + 2 e^{\frac{s^2}{2}} + c\right]$$

$e^{-x} = 1 - x +$

$$y(s) = \frac{1}{s} + \frac{2}{s^2} + \frac{c}{s^2} e^{-\frac{s^2}{2}}$$

$$= \frac{1}{s} + \frac{2}{s^2} + \frac{c}{s^2}\left(1 - \frac{s^2}{2} + \frac{s^4}{4} + \frac{s^6}{12} + \cdots\right]$$

$$= \frac{1}{s} + \frac{2}{s^2} + c\left[\frac{1}{2} - \underbrace{\cancel{\frac{s^2}{4}}}_{=0} + \cdots\right] \Rightarrow \mathcal{L}^{-1}(s^k) =$$

$y(t) = 1 + (c+2)t$ $\qquad y'(t) = c+2$ $\qquad y(0) = 2 = c+2$

$c = 0$

$$\boxed{y(t) = 1 + 2t} \Longleftarrow$$